Substitučná metóda

Základom tejto metódy je derivácia zloženej funkcie.

$$(f \circ g)'(x) = \{f[g(x)]\}' = f'[g(x)].g'(x)$$

integrovaním obidvoch strán dostaneme

$$\int \{f[g(x)]\}'\,dx = \int f'[g(x)].g'(x)\,dx$$

V. $\int f(x)dx = \int f(g(t)).g'(t)dt = f[g(x)] + c$

príklad:

$$\int \frac{1}{4x-2}\,dx = [*] \int \frac{1}{t}\frac{dt}{4} = \frac{1}{4}\int \frac{1}{t}\,dt = \frac{1}{4}.\ln|t| + c = \frac{\ln|4x-2|}{4} + c$$

$*$ $t = 4x - 2$

$\frac{dt}{dx} = 4$

$\frac{dt}{4} = dx$

$$\int \sin 7x\,dx = [*] \int \sin t\,\frac{dt}{7} = \frac{-\cos t}{7} + c = -\frac{\cos 7x}{7} + c$$

$*$ $t = 7x$

$\frac{dt}{dx} = 7$

$\frac{dt}{7} = dx$

$$\int e^{x+1}\,dx = [*] \int e^t\,dt = e^t + c = e^{x+1} + c$$

$*$ $t = x + 1$

$\frac{dt}{dx} = 1$

$dt = dx$

$$\int \mathrm{tg}\,x\,dx = \int \frac{\sin x}{\cos x}\,dx = [*] \int \frac{\sin x}{t}\frac{dt}{-\sin x} = -\int \frac{1}{t}\,dt = -\ln|t| + c = -\ln|\cos x| + c$$

$*$ $t = \cos x$

$\frac{dt}{dx} = -\sin x$

$\frac{dt}{-\sin x} = dx$

$$\int \mathrm{cotg}\,x\,dx = \int \frac{\cos x}{\sin x}\,dx = [*] \int \frac{\cos x}{t}\frac{dt}{\cos x} = \int \frac{1}{t}\,dt = \ln|t| + c = \ln|\sin x| + c$$

$*$ $t = \sin x$

$\frac{dt}{dx} = \cos x$

$\frac{dt}{\cos x} = dx$

$$\int \frac{x-1}{x+2}\,dx = \int \frac{x+2-3}{x+2}\,dx = \int \frac{x+2}{x+2} - \frac{3}{x+2}\,dx = \int 1 - \frac{3}{x+2}\,dx = \int 1\,dx - \int \frac{3}{x+2}\,dx =$$

$$= [*]\ x - \int \frac{3}{t}\,dt = x - 3.\ln|t| + c = x - 3\ln|x+2| + c$$

$*$ $t = x + 2$

$\frac{dt}{dx} = 1$

$dt = dx$

$$\int \frac{4x+3}{x-5}\,dx = \int \frac{4x-20+23}{x-5}\,dx = \int \frac{4x-20}{x-5} + \frac{23}{x-5}\,dx = \int \frac{4(x-5)}{x-5} + \frac{23}{x-5}\,dx = \int 4 + \frac{23}{x-5}\,dx =$$

$$= \int 4\,dx + \int \frac{23}{x-5}\,dx = [*]\ 4.x + \int \frac{23}{t}\,dt = 4x + 23.\ln|t| + c = 4x + 23\ln|x-5| + c$$

$*$ $t = x - 5$

$\frac{dt}{dx} = 1$

$dt = dx$

$$\int \frac{5x+7}{2x-1}dx = \int \frac{5x-2{,}5+9{,}5}{2x-1}dx = \int \frac{5x-2{,}5}{2x-1} + \frac{9{,}5}{2x-1}dx = \int \frac{2{,}5(2x-1)}{2x-1} + \frac{9{,}5}{2x-1}dx = \int \frac{5}{2} + \frac{19}{2(2x-1)}dx =$$

$$= \int \frac{5}{2}dx + \frac{19}{2}\int \frac{1}{2x-1}dx = [*]\ \frac{5}{2}.x + \frac{19}{2}\int \frac{1}{t}\frac{dt}{2} = \frac{5x}{2} + \frac{19}{4}.\ln|t| + c = \frac{5x}{2} + \frac{19}{4}\ln|2x-1| + c$$

* $t = 2x - 1$

$\frac{dt}{dx} = 2$

$\frac{dt}{2} = dx$

$$\int \frac{4x^2-6x-5}{x+3}dx = \int \frac{4x^2-6x-54+49}{x+3}dx = \int \frac{4x^2-6x-54}{x+3} + \frac{49}{x+3}dx = \int \frac{(x+3)(4x-18)}{x+3} + \frac{49}{x+3}dx =$$

$$= \int 4x - 18 + \frac{49}{x+3}dx = \int 4x - 18\,dx + \int \frac{49}{x+3}dx = [*]\ 4.\frac{x^2}{2} - 18.x + \int \frac{49}{t}dt = 2x^2 - 18x + 49.\ln|t| + c =$$

$$= 2x^2 - 18x + 49.\ln|x+3| + c$$

delíme: $(4x^2 - 6x - 5):(x + 3) = 4x - 18$ a zvyšok je $+49$

* $t = x + 3$

$\frac{dt}{dx} = 1$

$dt = dx$

$$\int \frac{3x+1}{2x^2-5x-3}dx =$$

$\frac{3x+1}{2x^2-5x-3} = \frac{3x+1}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3}$ kde $A, B \in \mathbb{R}$

$\frac{A}{2x+1} + \frac{B}{x-3} = \frac{A(x-3)+B(2x+1)}{(2x+1)(x-3)} = \frac{Ax-3A+2Bx+B}{(2x+1)(x-3)} = \frac{x(A+2B)+(-3A+B)}{(2x+1)(x-3)}$

$\frac{x(A+2B)+(-3A+B)}{(2x+1)(x-3)} = \frac{3x+1}{(2x+1)(x-3)}$

z toho vznikne sústava rovníc:

$(A + 2B).x = 3x \rightarrow A + 2B = 3$

$\underline{-3A + B = 1}$ /.(-2)

$A + 2B = 3$

$\underline{6A - 2B = -2}$ /I. + II.

$7A = 1$ /:7

$A = \frac{1}{7}$

$-3.\frac{1}{7} + B = 1$

$B = \frac{10}{7}$

$$\int \frac{3x+1}{2x^2-5x-3}dx = \int \frac{\frac{1}{7}}{2x+1} + \frac{\frac{10}{7}}{x-3}dx = \frac{1}{7}\int \frac{1}{2x+1}dx + \frac{10}{7}\int \frac{1}{x-3}dx = [*]\ \frac{1}{7}\int \frac{1}{t}\frac{dt}{2} + [\nabla]\ \frac{10}{7}\int \frac{1}{u}du =$$

$$= \frac{1}{14}.\ln|t| + \frac{10}{7}.\ln|u| + c = \frac{1}{14}\ln|2x+1| + \frac{10}{7}.\ln|x-3| + c$$

* $t = 2x + 1$ ∇ $u = x - 3$

$\frac{dt}{dx} = 2$ $\frac{du}{dx} = 1$

$\frac{dt}{2} = dx$ $du = dx$